4. Calculation of temperature increase for doubling CO_2 content

We have shown that an *instant doubling* of the CO_2 content increases the opacity (opaqueness) of the atmosphere to heat radiation reducing its emission to space by about 4 Watts/m^2. We need to calculate the rise in temperature needed to warm up the Earth's atmosphere to radiate an extra 4 Watts/m^2 to restore the Earth's energy balance. This disturbance to the Earth's energy balance is referred to as a climate forcing of 4 Watts/m^2. First let's calculate the temperature increase for instant doubling the atmospheric CO_2 content when there is no feedback.

We calculate ΔF for instant doubling of the atmospheric CO_2 content by setting C = $2C_0$ in Equation 32:

$$\Delta T = 1.66\ \mathit{ln}\ (C/C_0) = 1.66 \times 0.693 = 1.2^oC$$

$$Instant\ doubling\ of\ CO_2\ gives\ the\ classic\ result\ \Delta T_s = 1.2^o\ C$$

Let's now calculate the temperature increase for instant doubling the atmospheric CO_2 content when there is feedback from a change in water vapour opacity due to a change in temperature. The increased surface temperature from the instant doubling of CO_2 content allows an increased water vapour content by maintaining a constant relative humidity. According to the Clausius-Clapeyron Equation (36 below) the 1.2 ^oC increase in the Earth's surface temperature due to CO2 itself gives a full 8% increase in the amount of water vapour held at saturation in the warmer atmosphere or a 6.2% increase at 0.77 global average relative humidity. The extra water vapour opacity increases the overall absorption by water vapour itself raising the surface temperature further.

The opacity of water vapour is a function of the water vapour partial pressure (P) and is given[2] by:

$$\tau = 0.0126\ P^{0.503} \tag{34}$$

On differentiation we get:

$$\Delta\tau = 0.00634\ \frac{\Delta P}{P_0^{0.497}} \tag{35}$$

The water vapour partial pressure is a function of temperature and relative humidity[2]

$$P = H(P_0 e^{-\left(L/_{RT}\right)}) \tag{36}$$

where R = 8.3145 $Jmol^{-1}K^{-1}$, molar gas constant

L = 43655 $Jmol^{-1}$, latent heat per mole of water

P_0 = 1.4 X 1011 Pa, water vapour saturation constant

H = 0.77, global average relative humidity

Using Equation 36 and T = 288.15^{o}K we obtain P_0 = 1315.86 Pa. The extra water vapour contained by the warmer atmosphere raises the partial pressure. If P_1 and P_2 are the partial pressures at two temperatures T_1 and T_2 respectively, Equation 36 takes the form:

$$P_2 = P_1 \, e^{\left\{-\frac{L}{R}\left(\frac{1}{T_2}-\frac{1}{T_1}\right)\right\}} \tag{37}$$

Using Equation 37 let's calculate the increase in partial pressure when the temperature increases from 288.15 ^{o}K to 289.35 ^{o}K.

$$\begin{aligned} P_2 &= 1315.86 \, e^{\left\{-\frac{43655}{8.3145}\left(\frac{1}{289.35}-\frac{1}{288.15}\right)\right\}} \\ &= 1419.15 \, Pa \end{aligned}$$

$\Delta P = P_2 - P_1 = 103.30$ Pa

From Equation 35 we have:

$$\Delta\tau = 0.00634 \, \frac{103.3}{(1315.86)^{0.497}} = 0.01844$$

Now the extra water vapour does its own absorption raising the surface temperature further. From Equations 24 and 27 we have:

$\Delta T = 32.09 \, \Delta\tau = 0.5917^{o}$C

This further increase in surface temperature will cause another cycle of water vapour feedback and so on. The temperature converges to 290.54^{o}K after 12 cycles of the water vapour feedback loop as shown in Table 2 below:

Cycle	T_0	T_1	P_0	P_1	ΔP	Δτ	ΔT
1	288.15	289.35	1315.86	1419.15	103.3	0.01844	0.5917
2	289.35	289.94	1419.15	1472.69	53.54	0.00921	0.2954
3	289.94	290.24	1472.69	1500.09	27.4	0.00462	0.1484
4	290.24	290.39	1500.09	1514.02	13.93	0.00233	0.0748
5	290.39	290.46	1514.02	1521.09	7.065	0.00118	0.0377
6	290.46	290.5	1521.09	1524.67	3.577	0.00059	0.0191
7	290.5	290.52	1524.67	1526.48	1.81	0.0003	0.0096
8	290.52	290.53	1526.48	1527.39	0.915	0.00015	0.0049
9	290.53	290.53	1527.39	1527.85	0.463	7.7E-05	0.0025
10	290.53	290.53	1527.85	1528.09	0.234	3.9E-05	0.0012
11	290.53	290.54	1528.09	1528.21	0.118	2E-05	0.0006
12	290.54	290.54	1528.21	1528.27	0.06	9.9E-06	0.0003

Table 2 – Each row gives the calculations for one cycle of the positive feedback loop for water vapour. The temperature converges after 12 cycles giving 2.4^{o}C as the total surface temperature increase.

In general, the water vapour feedback induced by the initial CO_2 content increase will double the sensitivity of the global surface temperature.

Water vapour provides a fast feedback after a temperature increase. Ice sheet melting is a slow feedback but still positive. Fortunately, we can evaluate the climate sensitivity for all fast feedbacks from 20,000 years ago to the present from the ice core data as shown by Hansen**[5]**.

$$Climate\ sensitivity = \frac{Temperature\ change}{Climate\ forcing}$$

Ice sheets have trapped air bubbles of the atmosphere over that period. The ice core data show a total climate forcing of about 6.5 Watts/m^2 maintained an equilibrium temperature of about 5 Celsius degrees, giving a climate sensitivity of about 0.75. That is, we get about 3 degrees temperature increase for a 4 Watts/m^2 disturbance of the Earth's energy balance for instant doubling of CO_2 content. The resulting uncertainty is $0.5C^o$. This is a precise evaluation of climate sensitivity without using climate models[5].

5. References

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